

Name John P. Ansauer Key ID _____

Midterm Exam # 1 - 40 Points

The exam is closed book and closed notes. Please show your work step by step. Simple calculators may be used (no graphing or financial calculators and no cell phones)

You must show your work to receive full credit

Problem 1 (10 points)

You have four classes. Each requires 4, 3, 2, and 3 hours per week of work. Further, the credit hours for each class are 5, 4, 4, and 3, respectively. Calculate the correlation of hours of work and credit hours. These may help:

$$\hat{\sigma}_x^2 = \frac{1}{n-1} \sum_{i=1}^n (x_i - \hat{\mu}_x)^2 \quad \hat{\sigma}_{xy} = \frac{1}{n-1} \sum_{i=1}^n (x_i - \hat{\mu}_x)(y_i - \hat{\mu}_y)$$

$$\hat{\mu}_w = \frac{1}{4} (4 + 3 + 2 + 3) = \frac{12}{4} = 3$$

$$\hat{\mu}_c = \frac{1}{4} (5 + 4 + 4 + 3) = \frac{16}{4} = 4$$

$$\hat{\sigma}_w^2 = \frac{1}{3} ((4-3)^2 + (3-3)^2 + (2-3)^2 + (3-3)^2) = \frac{2}{3}$$

$$\hat{\sigma}_c^2 = \frac{1}{3} ((5-4)^2 + (4-4)^2 + (4-4)^2 + (3-4)^2) = \frac{2}{3}$$

$$\hat{\sigma}_{wc} = \frac{1}{3} ((4-3)(5-4) + (3-3)(4-4) + (2-3)(4-4) + (3-3)(3-4))$$

$$\hat{\sigma}_w = \frac{1}{3}$$

$$\hat{\rho}_{wc} = \frac{\hat{\sigma}_{wc}}{\hat{\sigma}_w \hat{\sigma}_c} = \frac{\frac{1}{3}}{\sqrt{\frac{2}{3}} \cdot \sqrt{\frac{2}{3}}}$$

$$\boxed{\frac{1}{2}}$$

Problem 2 (15 Points)

The average number of coffees consumed by Professor Spearot per day is characterized by a normal distribution with mean 4 and standard deviation 2.

- a. What is the probability that Professor Spearot drinks exactly 4 coffees per day? (5 points)

0 +5

- b. What is the probability that Professor Spearot drinks more than 8 coffees per day (Wooooo!!)? (5 points)

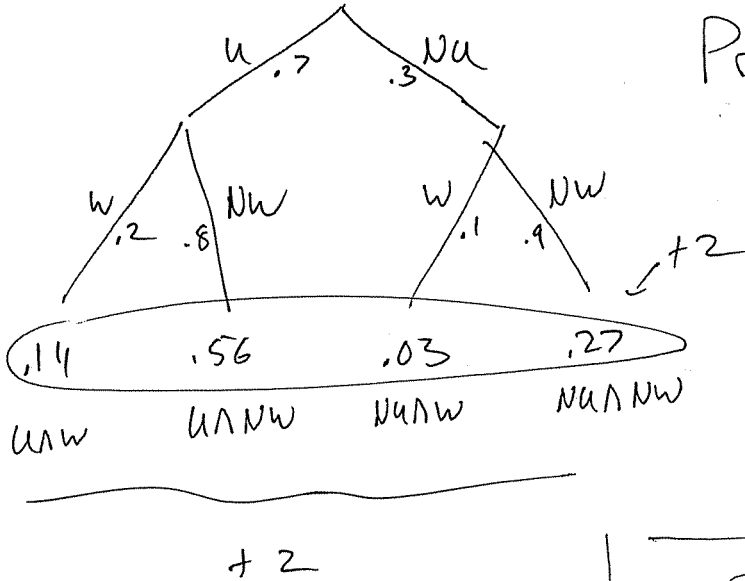
$$\begin{aligned} \Pr(C \geq 8) &= \Pr\left(Z \geq \frac{8-4}{2}\right) = \Pr(Z \geq 2) \\ &= 1 - \Pr(Z \leq 2) \\ +5 &= 1 - 0.9772 \\ &= \boxed{0.0228} \end{aligned}$$

- c. Professor Spearot actually only drinks 1 coffee per day. What is the probability that he consumes more than 1 coffee per day? (5 points)

$$\begin{aligned} \Pr(C \geq 1) &= \Pr\left(Z > \frac{1-4}{2}\right) = \Pr\left(Z > -\frac{3}{2}\right) \\ +5 &= \Pr\left(Z \leq \frac{3}{2}\right) \\ &= \boxed{0.9332} \end{aligned}$$

Problem 3 (15 Points)

a.) The Tigers are considering upgrading their bullpen for next year. The probability that they upgrade their bullpen is 0.7. If they upgrade their bullpen, the probability of winning the World Series is 0.2. If they do not upgrade their bullpen, the probability of winning the World Series is 0.1. Given that the Tigers don't win the World Series, what is the probability that they upgraded their bullpen before the season? (10 Points)



$$\begin{aligned}
 Pr(NW) &= Pr(u \cap NW) + Pr(nu \cap NW) \\
 &= 0.56 + 0.27 \\
 &= 0.83
 \end{aligned}$$

$$Pr(u|NW) = \frac{Pr(u \cap NW)}{Pr(NW)}$$

$$Pr(u|NW) = \frac{0.56}{0.83} = 0.675$$

b.) The Tigers earn 20 million if they win the World Series, and 0 if they don't. Upgrading the bullpen costs 3 million. Not upgrading the bullpen costs 0. What is the Expected Profit of the season? (5 Points)

$$\begin{aligned}
 E\Pi &= (20 - 3) \cdot 0.14 + (-3) \cdot (0.56) + 20 \cdot (0.03) + 0 \cdot 0.27 \\
 &= \boxed{1.3 \text{ Million}}
 \end{aligned}$$